Pseudo-Continuous Testing of the Latent Change Score Model

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Outline

The focus of this presentation will introduce the following model testing procedures:

1. Define regularization in the context of structural equation models
2. Pseudo-continuous fitting of the Dual Change Score model
3. Pseudo-continuous fitting of the Bivariate LCS model
4. Identification of time-varying effects in Bivariate LCS model

Regularized SEM

Applies both Ridge and Lasso penalties to various parameters in SEM (Jacobucci, Grimm, & McArdle, 2016).

$$F_{ML} = log(|\Sigma|) + tr(C * \Sigma^{-1}) - log(|C|) - p. \quad (1)$$

and builds in a separate element to the cost function

$$F_{regsem} = F_{ML} + \lambda P(\cdot) \quad (2)$$

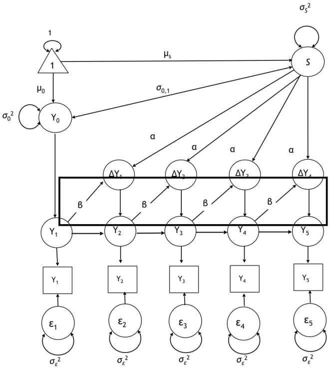
where λ is the regularization parameter, and takes some value between zero and infinity. When λ is zero, ML estimation is performed. $P(\cdot)$ is a general function for summing the values of one or more matrices. The two most common forms of $P(\cdot)$ include both the Lasso ($\|\cdot\|_1$) and Ridge ($\|\cdot\|_2$).

Bayesian Regularization

In Bayesian SEM, Normal distribution priors are equivalent to Ridge regularization (Lu, Chow, & Loken, 2016), while using Laplace distribution priors is a form of hybrid between Lasso and Ridge regularization (Jacobucci & Grimm, in preparation).

Dual Change Score Regularization

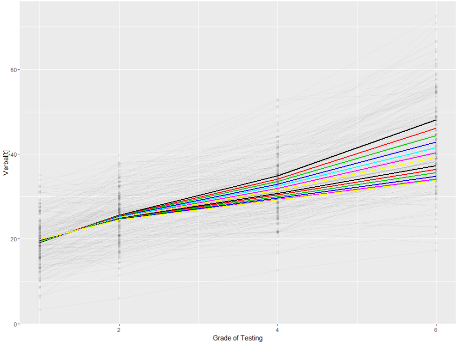
To test varying degrees of linearity, we can penalize the proportional change parameters



Example Using Verbal Scale from WISC Dataset

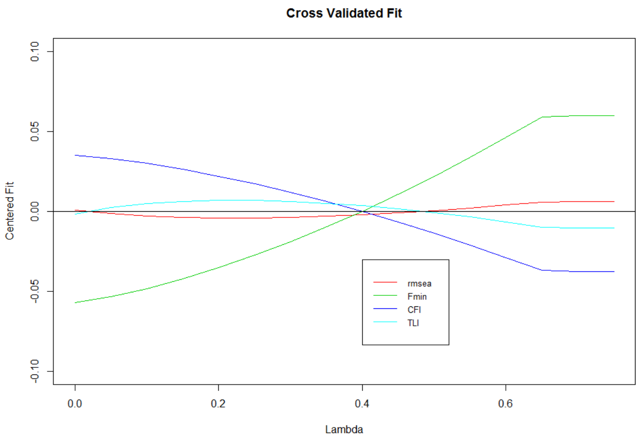
lambda	1 -> slope	dV2 -> dv1
0.00	-1.839	0.432
0.05	-1.069	0.381
0.10	-0.380	0.335
0.15	0.248	0.293
0.20	0.828	0.255
0.25	1.370	0.219
0.30	1.881	0.185
0.35	2.365	0.153
0.40	2.829	0.122
0.45	3.274	0.093
0.50	3.705	0.064
0.55	4.123	0.036
0.60	4.531	0.009
0.65	4.673	0.000
0.70	4.673	0.000

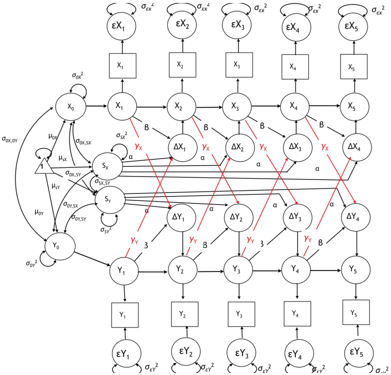
(a) Parameter estimates across 15 penalty values



(b) Expected mean trajectories

Cross-Validated Model Fit





Penalizing γ_x and γ_y allows the testing of no coupling through dual coupling models. When λ (penalty) is large, it is equal to the no coupling model. When $\lambda = 0$, it is the dual coupling modeling.

Time-Varying Parameters

Do the proportional change parameters vary across the time points?

Proposing a two-step process:

1. Run the model with parameters constrained across time
2. Run model while penalizing difference between freely estimated and constrained estimates

Example:

First step, coupling parameter estimates is estimated as 0.5 with all estimates constrained to be equal across time.
Second step. Start with large λ (penalty), placed on $\gamma_{11} - 0.5$. When λ is large, all parameters $=$ 0.5. When $\lambda = 0$, all coupling parameters are freely estimated.

Estimation Options

This pseudo-continuous form of LCS model testing can be accomplished with either frequentist or Bayesian estimation

Frequentist:

Using regsem package (Jacobucci, 2016) in R. Allows either Ridge or Lasso penalties to parameters from the A or S matrices in RAM notation.

Bayesian:

Place small variance Normal distribution (i.e. Ridge or Mplus's BSEM) or Laplace distribution (i.e. Lasso hybrid) on the parameters. Also, could do non-continuous test, and "search" for singular optimal model (Lu, Chow, & Loken, 2016).

Summary

Pairing either frequentist or Bayesian regularization with the Latent Change Score model allows for a more flexible approach to model testing.

Specifically:

1. Using cross-validation to inform model choice
2. Testing whether stationarity holds
3. Possibly improving convergence through constraining parameter estimates
4. A form of "automatic" model selection (i.e. Marcoulides & Ing, 2012)